

REVIEW of the book
Poncelet's Theorem

by

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This book is an Algebraic Geometry book. It deals with a famous result dating to the early part of the 19th century, namely the Poncelet's theorem. This theorem concerns conics and says that IF there is a n -sided polygon inscribed in an ellipse C and circumscribed about another ellipse D then for any point of C there is a n -sided polygon, having this point as a vertex, like above. The problem of that IF was solved by Cayley that theorem being also presented here. The Poncelet's theorem is a classic and important result since:

- it has a very simple formulation but is difficult to prove,
- there are generalized versions of this theorem (for instance, replace "ellipse" by "conic"),

- it relates to a large and diverse body of mathematics,
- it is a good opportunity for a less knowledgeable reader to see how diverse techniques, coming from different fields are used or to study them.

Let's elaborate on some of the assertions above. There are different proofs of this theorem none of which are elementary (Poncelet, Jacobi, Cayley, Griffiths and Harris etc.). The last two authors gave modern proofs using methods from Algebraic Geometry. They used complex analysis so, we come to the second assertion; there is a version of Poncelet's theorem in the complex projective plan. It seems that this particular aspect of this theorem, namely the revelation of deep connections between the seemingly disparate subjects, draws the author to a detailed study of this theorem and of its ramifications. The author has important contributions in queuing theory and in dynamical systems (billiards in an ellipse) so he is in a very good position to appreciate and explain these connections. This is, after all, the reason the book is so attractive.

The book consists of: preface, list of used symbols, an introductory chapter and four parts. In Chapter 1 we can read a comprehensive description of the main topic of the book. The topics presented in Part I and Part II are taken from various sources and contain fairly standard material, excepting Chapter 8 where is introduced the notion of division points on elliptic curves. Part I deals with projective geometry and Part II with complex analysis. The experienced reader could skip these parts and proceed to Part III and Part IV. These two parts form the core of the book. Part III discusses the theorems of Poncelet and Cayley from the point of view presented by Griffiths and Harris. Another approach, using notions from

dynamical systems, is presented here. Real version of the Poncelet's theorem is also proved in Part III. Part IV is called "Related Topics" and is particularly interesting. In Chapter 13 are discussed problems concerning mathematical billiards such as billiards in an ellipse. Chapter 14 deals with double queues and reveals unexpected connections with the main topic of the book. The supplementary Chapter 15 was written (based on a paper of S. B.) by Serge Tabachnikov and its title is "Billiards and the Poncelet's Theorem". Part IV also contains five Appendices that come to fill some omissions or to present some prerequisites.

The Bibliography is divided in two: Books and Papers. The book ends with an Index.

The material presented in this book should be understandable to anyone who has taken standard courses like: linear algebra, complex analysis, elementary topology. So, the book addresses to a large audience from students to mathematicians or, more general, to all who are interested in such "brilliant", as this one presented here, **The Poncelet's Theorem**.

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