

EXTREME RANKS OF (SKEW-)HERMITIAN SOLUTIONS TO A QUATERNION MATRIX EQUATION*

QING WEN WANG[†] AND JING JIANG[‡]

Abstract. The extreme ranks, i.e., the maximal and minimal ranks, are established for the general Hermitian solution as well as the general skew-Hermitian solution to the classical matrix equation $AXA^* + BYB^* = C$ over the quaternion algebra. Also given in this paper are the formulas of extreme ranks of real matrices $X_i, Y_i, i = 1, \dots, 4$, in a pair (skew-)Hermitian solution $X = X_1 + X_2i + X_3j + X_4k, Y = Y_1 + Y_2i + Y_3j + Y_4k$. Moreover, the necessary and sufficient conditions for the existence of a real (skew-)symmetric solution, a complex (skew-)Hermitian solution, and a pure imaginary (skew-)Hermitian solution to the matrix equation mentioned above are presented in this paper. Also established are expressions of such solutions to the equation when corresponding solvability conditions are satisfied. The findings of this paper widely extend the known results in the literature.

Key words. Quaternion matrix equation, Minimal rank, Maximal rank, Hermitian solution, Skew-Hermitian solution.

AMS subject classifications. 15A03, 15A09, 15A24, 15A33.

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[†]Department of Mathematics, Shanghai University, Shanghai 200444, P.R. China (wqw858@yahoo.com.cn). Supported by the grants from the Ph.D. Programs Foundation of Ministry of Education of China (20093108110001), the Scientific Research Innovation Foundation of Shanghai Municipal Education Commission (09YZ13), Natural Science Foundation of China (60672160), Singapore MoE Tier 1 Research Grant RG60/07, and Shanghai Leading Academic Discipline Project (J50101).

[‡]Department of Mathematics, Shanghai University, Shanghai 200444, P.R. China, and Department of Mathematics and Applied Mathematics, QiLu Normal University, Jinan 250013, P.R. China.