Erratum: Concentration bounds for stochastic approximations

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Abstract
We correct an error in [2] pointed out to us by Bernard Bercu.

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1 Introduction
We correct an error of the previous manuscript [2] gently pointed out to us by Bernard Bercu. We will freely use the notations of [2] and suppose the reader is familiar with them.

The error is the following: at the end of the proof of Proposition 5.2 we omitted to report a factor \( \sqrt{k} \) coming from the contribution \( E[|S_k|] \). The claim of the proposition should therefore be modified in the following way:

\[
\delta_n \leq \exp \left( -\lambda \Gamma_n \right) |\theta_0 - \theta^*| + [H]_1 \sigma_Y \left( \gamma_n h^{1/2} + \sum_{k=0}^{n-1} e^{-\Delta (\Gamma_n - \Gamma_{k+1})} \sqrt{E(\gamma_k - \gamma_{k+1})} + \sum_{k=1}^{n-1} e^{-\Delta (\Gamma_n - \Gamma_{k+1})} \sqrt{E\gamma_k \gamma_{k+1}} \right),
\]

where we recall \( \Gamma_n := \sum_{k=1}^{n} \gamma_k \), \( \sigma_Y := E \left[ F^2(Y) \right]^{1/2} < +\infty \), with \( F : y \mapsto E \left[ |y - Y| \right] \).

This does not affect the bound concerning the bias \( \delta_N \) in Theorem 2.2 when the step sequence \( (\gamma_n)_{n \geq 1} \) is s.t. \( \gamma_n = \frac{c}{n} \). However, when \( \gamma_n = \frac{c}{n^\rho} \), \( \rho \in (1/2,1) \) the above inequality does not provide a satisfactory control. In particular it gives an explosive bound for \( \rho \in (1/2,3/4] \). For \( \rho \in (3/4,1) \) it gives the convergence of \( \delta_n \) to 0 at a suboptimal rate.

2 Correct control of the bias in Proposition 5.2
This problem can be fixed by modifying Proposition 5.2. The new control writes:

\[
\forall n \geq 1, \ \delta_n \leq \exp \left( -\lambda \Gamma_n \right) |\theta_0 - \theta^*| + [H]_1 \sigma_Y \left( \sum_{k=0}^{n-1} e^{-2\Delta (\Gamma_n - \Gamma_{k+1})} \gamma_{k+1}^2 \right)^{1/2}. \tag{2.1}
\]

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To prove (2.1), we start from the identity $z_{n+1} = (I - \gamma_{n+1}J_n)z_n + \gamma_{n+1}\Delta M_{n+1}$ obtained in the proof of Proposition 5.2. Take now the square of the $L^2$-norm in the previous equality (instead of the $L^1$-norm as in [2]). Recalling that $\Delta M_{n+1}$ is a martingale increment, we derive:

$$
E[|z_{n+1}|^2] = E[(I - \gamma_{n+1}J_n)z_n|^2] + \gamma_{n+1}^2 E[|\Delta M_{n+1}|^2] \leq \exp(-2\lambda\gamma_{n+1})E[|z_n|^2] + \gamma_{n+1}^2[H]^2_1\sigma^2_Y.
$$

For the last inequality we used (exploiting assumption (HUA)) $||I - \gamma_{n+1}J_n|| \leq \exp(-\lambda\gamma_{n+1})$, $||.||$ standing for the matrix norm on $\mathbb{R}^d \otimes \mathbb{R}^d$, and the inequality $E[|\Delta M_{n+1}|^2] \leq [H]^2_1\sigma^2_Y$ which follows from (HL).

A direct induction yields for all $n \geq 1$:

$$
E[|z_n|^2] \leq \exp(-2\lambda \Gamma_n)|z_0|^2 + [H]^2_1\sigma^2_Y \left( \sum_{k=0}^{n-1} e^{-2\lambda(\Gamma_n - \Gamma_{k+1})} \gamma_{k+1}^2 \right).
$$

Eventually, equation (2.1) should be reported in Theorem 2.2. As already indicated, the associated bound for the bias when $\gamma_n = \frac{c}{n}$ writes

$$
\delta_N \leq \frac{|\theta_0 - \theta^*|}{N^{\Delta c^2}} + K[H]_1\sigma_Y \frac{1}{N^{\Delta c^2 / 2}}, \quad K := K(c),
$$

which remains the same as previously, whereas for $\gamma_n = \frac{c}{n^{1/2}}$, $\rho \in (1/2, 1)$ equation (2.1) yields

$$
\delta_N \leq \exp \left( -\frac{\Delta c}{1 - \rho} N^{1 - \rho} \right) |\theta_0 - \theta^*| + [H]_1\sigma_Y \frac{K}{N^{\Delta c^2 / 2}}, \quad K := K(c), \quad \forall \epsilon > 0,
$$

which also improves the formerly indicated bound. Let us also indicate that in the current framework, i.e under (HUA), (HL), computations similar to those leading to (2.1) can already be found in Duflo [1], see e.g. p. 56.

References
