Subgroups of $\mathbb{R}$-factorizable groups

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Abstract. The properties of $\mathbb{R}$-factorizable groups and their subgroups are studied. We show that a locally compact group $G$ is $\mathbb{R}$-factorizable if and only if $G$ is $\sigma$-compact. It is proved that a subgroup $H$ of an $\mathbb{R}$-factorizable group $G$ is $\mathbb{R}$-factorizable if and only if $H$ is $z$-embedded in $G$. Therefore, a subgroup of an $\mathbb{R}$-factorizable group need not be $\mathbb{R}$-factorizable, and we present a method for constructing non-$\mathbb{R}$-factorizable dense subgroups of a special class of $\mathbb{R}$-factorizable groups. Finally, we construct a closed $G_\delta$-subgroup of an $\mathbb{R}$-factorizable group which is not $\mathbb{R}$-factorizable.

Keywords: $\mathbb{R}$-factorizable group, $z$-embedded set, $\aleph_0$-bounded group, $P$-group, Lindelöf group

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1. Introduction

A topological group $G$ is called $\mathbb{R}$-factorizable ([7], [8]) if for every continuous function $g: G \to \mathbb{R}$ there exist a continuous homomorphism $\pi: G \to H$ of $G$ onto a second-countable topological group $H$ and a continuous function $h: H \to \mathbb{R}$ such that $g = h \circ \pi$. The reals $\mathbb{R}$ in this definition can be substituted by any second countable regular space $X$, thus giving us a possibility to factorize continuous functions $f: G \to X$ via continuous homomorphism onto second countable topological groups ([8]). The class of $\mathbb{R}$-factorizable groups is sufficiently wide; it contains all totally bounded groups, $\sigma$-compact groups (or, more generally, Lindelöf groups) and arbitrary subgroups of Lindelöf $\Sigma$-groups ([7], [8]). It is known, however, that subgroups of $\mathbb{R}$-factorizable groups do not inherit this property ([7, Example 2]).

In fact, some results on topological groups proved before 1990 can now be reformulated in terms of $\mathbb{R}$-factorizability. For example, the theorem proved on pages 118–119 of [6] is equivalent to say that every compact topological group is $\mathbb{R}$-factorizable. Theorem 1.2 of [2] implies, in particular, that every pseudocompact topological group is $\mathbb{R}$-factorizable. Note that every pseudocompact group is totally bounded ([2, Theorem 11]).

Our aim is to study $\mathbb{R}$-factorizable groups and their subgroups. We show first that a locally compact group is $\mathbb{R}$-factorizable if and only if it is $\sigma$-compact (Theorem 2.3). Then we characterize the subgroups of $\mathbb{R}$-factorizable groups which

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inherit this property: a subgroup $H$ of an $\mathbb{R}$-factorizable group $G$ is $\mathbb{R}$-factorizable if and only if $H$ is $z$-embedded in $G$ (Theorem 2.4). A slight modification of a construction in [7] gives us a lot of dense subgroups of $\mathbb{R}$-factorizable groups which are not $\mathbb{R}$-factorizable (see Theorem 3.1). We also construct a closed $G_5$-subgroup of an Abelian $\mathbb{R}$-factorizable group which is not $\mathbb{R}$-factorizable (Example 3.2).

Finally, we consider a formally weaker notion of a semi-$\mathbb{R}$-factorizable group and show that every semi-$\mathbb{R}$-factorizable group is $\mathbb{R}$-factorizable.

2. $z$-embedded subgroups of topological groups

The notion of an $\aleph_0$-bounded topological group introduced by Guran ([3]) plays an important rôle in our considerations.

**Definition 2.1.** A topological group $G$ is said to be $\aleph_0$-bounded if for each neighborhood $U$ of the identity, there exists a countable subset $M \subseteq G$ such that $G = M \cdot U$.

It is known ([3]) that a topological group $G$ is $\aleph_0$-bounded if and only if it embeds into a cartesian product of second countable topological groups as a topological subgroup. Although the following result was mentioned in [8], its proof was only sketched there.

**Lemma 2.2.** Every $\mathbb{R}$-factorizable group is $\aleph_0$-bounded.

**Proof:** Let $G$ be an $\mathbb{R}$-factorizable group. It suffices to show that $G$ can be embedded as a topological subgroup into a product of second countable groups. Let $N(e)$ be a neighborhood base at the identity $e$ of $G$. For every neighborhood $U \in N(e)$, let $f_U: G \to \mathbb{R}$ be a continuous function such that $f(e) = 1$ and $f(G \setminus U) = \{0\}$. Since $G$ is $\mathbb{R}$-factorizable, there exist a second countable group $H_U$, a continuous homomorphism $\pi_U: G \to H_U$ and a continuous function $h: H_U \to \mathbb{R}$ such that $f = h \circ \pi_U$. Observe that the diagonal product $\varphi = \Delta\{\pi_U : U \in N(e)\}$ is a topological monomorphism of $G$ to the group $\Pi = \prod\{H_U : U \in N(e)\}$.

Since second countable groups $H_U$ are $\aleph_0$-bounded, the group $\Pi$ is $\aleph_0$-bounded as well. Now, subgroups of $\aleph_0$-bounded groups are $\aleph_0$-bounded, so $G$ inherits this property. \qed

**Theorem 2.3.** A locally compact $\mathbb{R}$-factorizable group is $\sigma$-compact.

**Proof:** Suppose that $G$ is a locally compact $\mathbb{R}$-factorizable group. Then there exists a neighborhood $U$ of the identity of $G$ such that $\overline{U}$ is compact. Since every $\mathbb{R}$-factorizable group is $\aleph_0$-bounded (Lemma 2.2), there is a countable subset $C \subseteq G$ such that $C \cdot U = G$. Therefore, $\{g \cdot \overline{U} : g \in C\}$ is a countable family of compact sets whose union is $G$. \qed

Tkačenko [7] showed that subgroups of $\mathbb{R}$-factorizable groups are not necessarily $\mathbb{R}$-factorizable. On the other hand, an $\mathbb{R}$-factorizable subgroup of an arbitrary topological group $G$ is $z$-embedded in $G$ ([4]). In the following theorem we give
a complete characterization of subgroups of $\mathbb{R}$-factorizable groups which preserve the property of $\mathbb{R}$-factorizability. Let $X$ be a topological space and let be $A \subseteq X$. We say that $A$ is $z$-embedded in $X$ if every cozero set $B$ in $A$ is of the form $B = A \cap C$, where $C$ is a cozero set in $X$.

**Theorem 2.4.** A subgroup $H$ of an $\mathbb{R}$-factorizable group $G$ is $\mathbb{R}$-factorizable if and only if $H$ is $z$-embedded in $G$.

**Proof:** We shall only give the proof of the fact that $z$-embedding is a sufficient condition for the subgroup $H$ to be $\mathbb{R}$-factorizable because the proof of necessity appears as Theorem 3.1 of [4]. Let $f: H \to \mathbb{R}$ be a continuous function. Consider the family $\gamma$ of all open intervals in $\mathbb{R}$ with rational end points. For every $U \in \gamma$, let $V_U$ be a cozero set in $G$ such that $V_U \cap H = f^{-1}(U)$. There exists a continuous function $g_U: G \to \mathbb{R}$ such that $g_U(U) = V_U$. The diagonal product $g = \Delta_{U \in \gamma} g_U$ is a continuous mapping of $G$ to the second countable space $\mathbb{R}\gamma$ and, by $\mathbb{R}$-factorizability of $G$, there exist a continuous homomorphism $\pi$ of $G$ onto a second countable topological group $G^*$ and a continuous function $g^*: G^* \to \mathbb{R}\gamma$ such that $g = g^* \circ \pi$.

![Diagram 1](image)

We claim that for any $x_0, x_1 \in H$, $f(x_0) = f(x_1)$ whenever $\pi(x_0) = \pi(x_1)$. Assume the contrary, let $f(x_0) \neq f(x_1)$ for some $x_0, x_1 \in H$ with $\pi(x_0) = \pi(x_1)$. We can also assume that $f(x_0) < f(x_1)$. If $r_0, r_1$ and $r_2$ are rationals and $r_0 < f(x_0) < r_1 < f(x_1) < r_2$, consider the intervals $U_0 = (r_0, r_1) \in \gamma$ and $U_1 = (r_1, r_2) \in \gamma$. Let $p_{U_0}: \mathbb{R}\gamma \to \mathbb{R} = \mathbb{R}_{U_0}$ be the natural projections, $g \circ p_{U_0} = g_{U_0} (i = 0, 1)$. On the one hand, the sets $g_{U_0}^{-1}(U_0) \cap H = f^{-1}(U_0)$ and $g_{U_1}^{-1}(U_1) \cap H = f^{-1}(U_1)$ are disjoint. This is equivalent to say that $g^{-1}(O_0) \cap H$ and $g^{-1}(O_1) \cap H$ are disjoint, where $O_i = p_{U_i}^{-1}(U_i) \ni g(x_i)$ ($i = 0, 1$). In particular, $g(x_0) \neq g(x_1)$. On the other hand, $g = g^* \circ \pi$, whence $g(x_0) = g(x_1)$, a contradiction.

Put $H^* = \pi(H)$. The assertion just proved implies that there exists a function $g_*: H^* \to \mathbb{R}$ such that $f = g_* \circ \pi |_H$. It remains to verify that $g_*$ is continuous. Let $U \in \gamma$ be arbitrary. Then

$$g_*^{-1}(U) = \pi(f^{-1}(U)) = \pi(g_U^{-1}(U) \cap H) = (g^*)^{-1}(p_{U_0}^{-1}(U)) \cap \pi(H)$$

is open in $\pi(H) = H^*$. Since $\gamma$ is a base for $\mathbb{R}$, this proves the continuity of $g_*$. Thus, we have $f = g_* \circ \varphi$, where $\varphi = \pi |_H$ is a continuous homomorphism of $H$ onto the second countable group $H^* \subseteq G^*$, and hence $H$ is $\mathbb{R}$-factorizable. \qed
It is clear that every retract of a space $X$ is $z$-embedded in $X$. Indeed, if $r: X \to X$ is a retraction and $Y = r(X)$, then for each continuous function $f: Y \to \mathbb{R}$, the function $\hat{f} = f \circ r$ is a continuous extension of $f$ to $X$. Note also that if $G$ is a topological group and $H$ is an open subgroup of $G$, then $H$ is a retract of $G$. Indeed, in every left coset $U$ of $H$ in $G$, pick a point $x_U \in U$. Define $r: G \to H$ in the following way: if $g \in H$, then $f(g) = g$; if $g \in U$ and $U \neq H$, put $r(g) = x_U^{-1}g$. Since the left cosets are open and disjoint, the continuity of $r$ is immediate. From these two observations we deduce the following results.

**Corollary 2.5.** Let $G$ be an $\mathbb{R}$-factorizable group and $H$ a subgroup of $G$. If $H$ is a retract of $G$, then $H$ is $\mathbb{R}$-factorizable.

**Corollary 2.6.** An open subgroup of an $\mathbb{R}$-factorizable group is $\mathbb{R}$-factorizable.

### 3. Some examples

By Corollary 1.13 of [8], every Lindelöf topological group is $\mathbb{R}$-factorizable. Let us call a topological group $G$ a $P$-group if any intersection of countably many open sets in $G$ is open. Making use of the existence of a special Lindelöf $P$-group $\hat{G}$ of weight $\aleph_1$ (see [1]), Tkačenko [7] constructed an example of a proper dense subgroup of $\hat{G}$ which was not $\mathbb{R}$-factorizable. Our aim is to show that any proper dense subgroup of an arbitrary Lindelöf $P$-group of weight $\aleph_1$ is not $\mathbb{R}$-factorizable.

**Theorem 3.1.** If $H$ is a proper dense subgroup of a Lindelöf $P$-group $G$ of weight $\aleph_1$, then $H$ is not $\mathbb{R}$-factorizable.

**Proof:** Since $G$ is a $P$-group, it is zero-dimensional. Therefore, we choose a base $\mathcal{B} = \{O_\alpha : \alpha < \omega_1\}$ at the identity $e$ of $G$ satisfying the following conditions for each $\alpha < \omega_1$:

1. $O_\alpha$ is a clopen set;
2. $O_\alpha = \bigcap_{\beta < \alpha} O_\beta$ for any limit ordinal $\alpha < \omega_1$;
3. $O_{\alpha+1} = O_\alpha \cup B_\alpha$ where $A_\alpha$ and $B_\alpha$ are nonempty disjoint clopen sets.

Now define $U'$ and $V'$ by $U' = (G \setminus O_0) \cup (\bigcup_{\alpha < \omega_1} A_\alpha)$ and $V' = \bigcup_{\alpha < \omega_1} B_\alpha$. From conditions (1) and (4) it follows that $U'$ and $V'$ are open sets. Conditions (2) and (4) imply that $U' \cup V' = G \setminus \{e\}$. Finally, (3) guarantees that $U'$ and $V'$ are nonempty.

Pick a point $g \in G \setminus H$ and define $U = gU' \cap H$ and $V = gV' \cap H$. Then $U$ and $V$ are non-empty open subsets of $H$ and $H = U \cup V$. Let $f$ be the function on $H$ defined by the rule $f(x) = 0$ if $x \in U \cap H$ and $f(x) = 1$ if $x \in V \cap H$. It is easy to see that $f$ is continuous. Let $\pi: H \to K$ be a continuous homomorphism of $H$ to a metrizable group $K$. Then the kernel of $\pi$ is a $G_\delta$-set in $H$, and hence is an open neighborhood of $e$. So, we can find $\alpha < \omega_1$ such that $O_\alpha \cap H \subseteq \ker \pi$. Pick points $a \in H \cap gA_{\alpha+1}$ and $b \in H \cap gB_{\alpha+1}$. Then $ab^{-1} \in O_\alpha$ by (3) and...
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The above theorem shows that there are many subgroups of $\mathbb{R}$-factorizable groups which are not $\mathbb{R}$-factorizable. In special classes of $\mathbb{R}$-factorizable groups the situation changes: by Corollary 1.13 of [8], every subgroup of a $\sigma$-compact topological group is $\mathbb{R}$-factorizable. Intuitively, $G_\delta$-subgroups of a topological group seem close to be $z$-embedded in it. Thus, Theorem 2.4 might suggest the conjecture that a closed $G_\delta$-subgroup of an $\mathbb{R}$-factorizable group is $\mathbb{R}$-factorizable as well. We show below that this is not the case.

Example 3.2. Let $H$ be an $\aleph_0$-bounded Abelian group of weight $\aleph_1$ which is not $\mathbb{R}$-factorizable ([7, Example 2.1]). By a theorem of Guran [3], $H$ can be considered as a subgroup of a product $\Pi = \prod_{\alpha<\omega_1} G_{\alpha}$, where each $G_{\alpha}$ is a second countable Abelian group. Let $G = \Pi^\omega$. The subgroup $H'$ of $G$ that consists of all elements of the form $(h,h,...)$ with $h \in H$ is isomorphic to $H$.

By the Hewitt–Marczewski–Pondiczery theorem there exists a countable dense subset $S$ of $\Pi$. Consider the subset $D$ of $G$ of all elements $x \in G$ such that for a finite set of $n_1,\ldots,n_k \in \omega$, $x(n_i) \in S$ and $x(n) = 0$ for other indices $n$. It is easy to see that the set $D$ is countable and dense in $G$. Let $K = \langle D \rangle$ be the subgroup of $G$ generated by $D$. Then $K$ is a countable dense subgroup of $G$ and $K \cap H' = \{e_G\}$. Since any dense subgroup of a product of second countable groups is $\mathbb{R}$-factorizable ([8, Corollary 1.10]), we conclude that the subgroup $L = K + H'$ of $G$ is $\mathbb{R}$-factorizable. On the other hand, since the diagonal $\Delta = \{(x,x,\ldots) : x \in G\}$ of the group $G = \Pi^\omega$ is closed in $G$ and $H' \subseteq \Delta$, we have $\overline{H'} \subseteq \Delta$ and $\Delta \cap K = \{e_G\}$, whence $\overline{H'} \cap L = H'$. This means that $H'$ is a closed subgroup of $L$. For each $x \in K$, $x + H'$ is a closed subset of $L$ and it is easy to see that

$$H' = \bigcap_{x \in K \setminus \{e_G\}} L \setminus (x + H').$$

Hence, $H' \simeq H$ is a closed $G_\delta$-subgroup of the $\mathbb{R}$-factorizable group $L = K + H'$, which is not $\mathbb{R}$-factorizable.

4. Semi-$\mathbb{R}$-factorizable groups

The fact that a topological group $G$ is $\mathbb{R}$-factorizable can be expressed in the following form equivalent to the original one: given a continuous function $f: G \to \mathbb{R}$, there exist a closed normal subgroup $H$ of $G$, a Hausdorff second countable group topology $\tau$ for the quotient group $G/H$ coarser than the quotient topology $\tau_\pi$ and a continuous function $h: (G/H, \tau) \to \mathbb{R}$ such that $f = h \circ \pi$, where $\pi: G \to G/H$ is the quotient homomorphism.

The motivation of the definition below arises if one omits the condition of normality of the subgroup $H \subseteq G$. Thus, we define a class of topological groups
containing $\mathbb{R}$-factorizable groups. We will see, however, that the two classes coincide (Theorem 4.3).

Let $H$ be a closed subgroup of a topological group $G$ and $G/H = \{xH : x \in G\}$ a left coset space with the quotient topology $\tau_q$. A topology $\tau \subseteq \tau_q$ for $G/H$ is called left-invariant if the functions $\phi_a: G/H \to G/H$ defined by $\phi_a(xH) = axH$, $x \in G$, are continuous for all $a \in G$. This notation will be used in the proofs of Lemma 4.2 and Theorem 4.3.

**Definition 4.1.** A topological group $G$ is said to be semi-$\mathbb{R}$-factorizable provided that for every continuous function $f: G \to \mathbb{R}$ there exists a closed subgroup $H$ of $G$, a second countable left-invariant $T_1$ topology $\tau$ on the left coset space $G/H$ coarser than the quotient topology and a continuous function $h: (G/H, \tau) \to \mathbb{R}$ such that $f = h \circ \pi$, where $\pi: G \to G/H$ is the natural projection.

**Lemma 4.2.** Every semi-$\mathbb{R}$-factorizable group is $\aleph_0$-bounded.

**Proof:** Let $G$ be a semi-$\mathbb{R}$-factorizable group and $V$ an open neighborhood of the identity $e$ in $G$. Since a topological group is completely regular, there exists a continuous function $f: G \to [0,1]$ such that $f(e) = 1$ and $f(G \setminus V) = \{0\}$. Since $G$ is semi-$\mathbb{R}$-factorizable, there exists a closed subgroup $H$ of $G$, a left-invariant second countable $T_1$ topology $\tau$ on $G/H$ and a continuous function $h: (G/H, \tau) \to \mathbb{R}$ such that $f = h \circ \pi$, where $\pi: G \to G/H$ is the natural projection. The set $U = h^{-1}(\frac{1}{2},1]$ is open in $(G/H, \tau)$ and $e \in \pi^{-1}(h^{-1}(\frac{1}{2},1]) = f^{-1}(\frac{1}{2},1] \subseteq V$. For each $g \in G$, the function $\sigma_g: G \to G$ defined by $\sigma_g(x) = gx$ is a homeomorphism of $G$ onto $G$. Note that $\pi \circ \sigma_g = \phi_g \circ \pi$ and, therefore, $f \circ \sigma_g = h \circ \pi \circ \phi_g = h \circ \phi_g \circ \pi$. Since

$$(f \circ \sigma_{x^{-1}})^{-1}(\frac{1}{2},1] = \sigma_{x^{-1}}^{-1}(f^{-1}(\frac{1}{2},1]) = \sigma_x(f^{-1}(\frac{1}{2},1]) \subseteq \sigma_x(V) = xV,$$

we conclude that $U_x = \phi_{x^{-1}}^{-1}(h^{-1}(\frac{1}{2},1])$ is open in $(G/H, \tau)$ and $\pi^{-1}(U_x) \subseteq xV$. The collection $\{U_x : x \in G\}$ covers $G/H$. Since $G/H$ has countable weight, there exists a sequence $x_0, x_1, \ldots$ of elements of $G$ such that $G/H \subseteq \bigcup_{i=0}^{\infty} U_{x_i}$. Consequently, the family $\{\pi^{-1}(U_{x_i}) : i \in \omega\}$ covers $G$ and, therefore, the corresponding family $\{x_i V : i \in \omega\}$ also covers $G$. This proves that $G$ is $\aleph_0$-bounded. $\square$

**Theorem 4.3.** Every semi-$\mathbb{R}$-factorizable group is $\mathbb{R}$-factorizable.

**Proof:** Let $G$ be a semi-$\mathbb{R}$-factorizable group and $f: G \to \mathbb{R}$ a continuous function. Then $G$ has a closed subgroup $H$ such that there exist a left-invariant second countable $T_1$ topology $\tau$ on $G/H$ and a continuous function $h: (G/H, \tau) \to \mathbb{R}$ such that $f = h \circ \pi$, where $\pi: G \to G/H$ is the natural projection. If $\{W_i : i \in \omega\}$ is a local base of $G/H$ at $\{H\}$, then $H = \bigcap_{i \in \omega} \pi^{-1}(W_i)$. Since $G$ is $\aleph_0$-bounded (Lemma 4.2), for every $U_i = \pi^{-1}(W_i)$ there exist a continuous homomorphism $\pi_i: G \to H_i$ of $G$ onto a second countable group $H_i$ and a neighborhood $V_i$ of the identity in $H_i$ such that $\pi_i^{-1}(V_i) \subseteq U_i$ (see [3]). Then $N = \bigcap_{i \in \omega} \ker \pi_i$ is a closed normal subgroup of $G$ and $N \subseteq H$. First, we define a second countable
group topology $t$ for $G/N$. Let $\varphi_i: G/N \to H_i$ be the homomorphism defined by $\varphi_i(aN) = \pi_i(a)$, $a \in G$. Note that $\varphi_i$ is well-defined because if $b \in aN$ then $a^{-1}b \in N \subseteq \ker \pi_i$, and hence $\pi_i(a) = \pi_i(b)$. Let $t$ be the weakest group topology on $G/N$ that makes each of the homomorphisms $\varphi_i$ continuous. It is clear that $(G/N, t)$ is a topological group because the topology $t$ is generated by a family of homomorphisms, and $t$ is second countable because each group $H_i$ is second countable. We define the function $\tilde{h}: G/N \to \mathbb{R}$ by $\tilde{h}(aN) = h(aH)$, i.e., $\tilde{h} = h \circ \psi$, where $\psi: G/N \to G/H$ is given by $\psi(aN) = aH$. It is easy to see that $\psi$ is well-defined because the left cosets of $N$ in $G$ are contained in the left cosets of $H$ in $G$. Let $\pi_N$ be the natural projection of $G$ onto $G/N$. Then $\tilde{h} \circ \pi_N = h \circ \pi = f$ (see Diagram 2 below).

Finally, we have to prove that the function $\tilde{h}$ is continuous. To this end, it suffices to show that $\psi$ is continuous, that is, for each $A \in G/N$ and each open set $V \in \tau$ containing $\psi(A)$, there exists $U \in t$ with $A \in U$ such that $\psi(U) \subseteq V$. Since $A = gN$ for some $g \in G$, it follows from the definition of $\psi$ that $\psi(A) = gH$. Since the topology $\tau$ on $G/H$ is left-invariant, the set $V$ has the form $\phi_g(V')$, where $H \in V' \subseteq \tau$. There exists $i \in \omega$ such that $W_i \subseteq V'$. Recall that $\pi_i^{-1}(V_i) \subseteq U_i = \pi^{-1}(W_i)$ by the choice of the neighborhood $V_i$ of the identity in $H_i$. Define $O = \varphi_i^{-1}(V_i)$ and $U = a \cdot O$, where $a = \pi_N(g)$. Then $A \in U \in t$ and

$$
\psi(U) = \psi(a \cdot O) = \pi(g \cdot \pi_i^{-1}(V_i)) = \phi_g(\pi(\pi_i(V_i))) \\
\subseteq \phi_g(\pi(U_i)) \subseteq \phi_g(\pi \pi^{-1}(W_i)) = \phi_g(W_i) \subseteq \phi_g(V') = V.
$$

This implies the continuity of $\psi$, and hence the function $\tilde{h} = h \circ \psi$ is continuous as well.

**References**


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