Counterexample to boundary regularity of a strongly pseudoconvex CR submanifold: An addendum to the paper of Harvey-Lawson

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The purpose of this paper is to give a counterexample of Theorem 10.4 in [Ha-La]. In the Harvey-Lawson paper, a global result is claimed, but only a local result is proven. This theorem has had a big impact on CR geometry for almost a quarter of a century because one can use the theory of isolated singularities to study the theory of CR manifolds and vice versa.

Example. Consider the following holomorphic map:

\[ F : \mathbb{C}^2 \rightarrow \mathbb{C}^3 \]

\[ (u, v) \rightarrow (x, y, z) = \left( u(u - 1), v, u^2(u - 1) \right). \]

Clearly for any \( c \), \( F \) restricted on the line \( \{ v = c \} \) is an embedding outside the two points \( (0, c) \) and \( (1, c) \). \( F \) sends \( (0, t) \) and \( (1, t) \) to \( (0, t, 0) \) for all \( t \). Now take \( S \), which is the boundary of a ball \( B = \{ (u, v) \in \mathbb{C}^2 : \| (u, v) \| \leq 2 \} \). It is easy to see that the mapping \( F \) restricted on \( S \) is still an embedding. The image of \( S \) under \( F \) is a strongly pseudoconvex CR manifold in \( \mathbb{C}^3 \). The variety that \( F(S) \) bounds is \( F(B) \). Observe that \( F(B) \) has curve singularities along the line \( (0, t, 0) \). We remark that \( F(\mathbb{C}^2) \) is a hypersurface \( \{ (x, y, z) \in \mathbb{C}^3 : z^2 - zx - x^3 = 0 \} \) in \( \mathbb{C}^3 \).

Theorem 10.4 of [Ha-La] was so powerful that it has been used by many researchers. Fortunately, we can replace it by the following theorem, the proof of which will appear elsewhere [Lu-Ya].

Theorem. Let \( X \) be a strongly pseudoconvex CR manifold of dimension \( 2n - 1 \), \( n \geq 2 \). If \( X \) is contained in the boundary of a bounded strictly pseudoconvex domain \( D \) in \( \mathbb{C}^N \), then there exists a complex analytic subvariety \( V \) of dimension \( n \) in \( D - X \) such that the boundary of \( V \) is \( X \). Moreover, \( V \) has boundary regularity at every point of \( X \), and \( V \) has only isolated singularities in \( V|X \).

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