A note on “The nearest symmetric fuzzy solution for a symmetric fuzzy linear system”

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Abstract

This paper provides accurate approximate solutions for the symmetric fuzzy linear systems in (Allahviranloo et al. [1]).

1 Introduction

The following section reviews basic definitions of fuzzy theory, which will be needed in the sequel:

**Definition 1.1.** Let $X$ be a universal set. Then, we define the fuzzy subset $	ilde{A}$ of $X$ by its membership function $\mu_{\tilde{A}} : X \to [0,1]$ which assigns to each element $x \in X$ a real number $\mu_{\tilde{A}}(x)$ in the interval $[0,1]$; where the value $\mu_{\tilde{A}}(x)$ represents the grade of membership of $x$ in $\tilde{A}$. A fuzzy set $\tilde{A}$ is written as:

$$\tilde{A} = \{(x, \mu_{\tilde{A}}(x)), x \in X, \mu_{\tilde{A}}(x) \in [0,1]\}.$$

**Definition 1.2.** A fuzzy set $\tilde{A}$ in $X = \mathbb{R}^n$ is convex fuzzy set if:

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∀x₁, x₂ ∈ X, ∀λ ∈ [0, 1],
µ̃A(λx₁ + (1 − λ)x₂) ≥ min(µ̃A(x₁), µ̃A(x₂)).

**Definition 1.3.** Let ̃A be a fuzzy set defined on the set of real numbers R. ̃A is called normal fuzzy set if there exist x ∈ R such that µ̃A(x) = 1.

**Definition 1.4.** A fuzzy number is a normal and convex fuzzy set, with its membership function µ̃A(x) defined in real line R and piecewise continuous.

**Definition 1.5.** A fuzzy number ̃A = (a₁, a₂; α, β)LR is said to be an L-R fuzzy number, where its membership function satisfy

\[
µ̃A(x) = \begin{cases} 
L(\frac{a_1 - x}{α}), & x ≤ a_1, \quad α > 0, \\
1, & a_1 ≤ x ≤ a_2, \\
R(\frac{x - a_2}{β}), & a_2 ≤ x, \quad β > 0.
\end{cases}
\]

Where a₁ ≤ a₂, and α and β are the left and right spreads, respectively; and the functions L(\(.), R(\(.)\)), which are called left and right shape function, satisfying:

1. L(\(.), R(\(.)\)) are non-increasing from \(\mathbb{R}^+\) to \([0, 1]\),
2. L(0) = R(0) = 1, L(1) = R(1) = 0.

Also, if α = β and L(x) = R(x) for all x ∈ R we say ̃A is a symmetric L-L fuzzy number.

**Definition 1.6.** (Allahviranloo et al.[1]) Let the shape functions L(\(\).\), R(\(\).\)) are fixed. Consider two L-R fuzzy numbers as ̃A = (a₁, a₂; α, β), and ̃B = (b₁, b₂; γ, η). We define the distance between ̃A and ̃B as follows:

\[
d( ̃A, ̃B) = \sqrt{\frac{1}{4}[(a_1 - b_1)^2 + (a_2 - b_2)^2 + (α - γ)^2 + (β - η)^2 + (a_1 - a_2)^2 + (a_2 - b_2)^2]}.\]

**Definition 1.7.** A vector ̃X = (̃x₁, ̃x₂, ..., ̃xₙ), where ̃xᵢ, 1 ≤ i ≤ n are L-R fuzzy numbers, is called an L-R fuzzy vector.

**Definition 1.8.** (Allahviranloo et al.[1]) For two L-R fuzzy vectors ̃X = (̃x₁, ̃x₂, ..., ̃xₙ), ̃Y = (̃y₁, ̃y₂, ..., ̃yₙ) we defined

\[
D_p( ̃X, ̃Y) = \left(\sum_{i=1}^{n} d_p( ̃x_i, ̃y_i)\right)^{\frac{1}{p}}.
\]

as distance between them, where p ≥ 1.
2 Numerical examples

In this section we provide proposed solutions for the examples in [1].

**Example 2.1.** (Allahviranloo et al.[1])
According to [1], the symmetric exact solution for $S$-L-FLS is:

$$\tilde{X}_v = \begin{bmatrix} (x_1^1, x_2^1; \alpha_1^x, \alpha_1^y) \\ (x_1^2, x_2^2; \alpha_2^x, \alpha_2^y) \\ (x_1^3, x_2^3; \alpha_3^x, \alpha_3^y) \end{bmatrix} = \begin{bmatrix} (1, 2; 2, 2) \\ (-1, 1; 1, 1) \\ (2, 3; 3, 3) \end{bmatrix}.$$

But $\tilde{X}_v$ does not correspond to the system, for instance if $b_1^1 = 2$ is examined in vector $\tilde{B}$, we get:

$$(-1)(2) + (-1)(1) + (1)(2) = -1.$$

By using Definition 1.8. we produce $D_2\left(A\tilde{X}_v, \tilde{B}\right) = D_1\left(A\tilde{X}_v, \tilde{B}\right) = 3$.

However, the symmetric exact solution corresponds to the system by solving the associated linear system is as follows:

$$\tilde{X}_e = \begin{bmatrix} (-\frac{5}{4}, -\frac{1}{4}; 2, 2) \\ (-7, -5; 1, 1) \\ (-\frac{13}{4}, -\frac{5}{4}; 3, 3) \end{bmatrix},$$

$$D_p\left(A\tilde{X}_e, \tilde{B}\right) = 0, \forall p \geq 1.$$

**Example 2.2** (Allahviranloo et al.[1])
According to[1], the nearest symmetric approximate solution is:

$$\tilde{X}_v = \begin{bmatrix} (x_1^1, x_2^1; \alpha_1^x, \alpha_1^y) \\ (x_1^2, x_2^2; \alpha_2^x, \alpha_2^y) \\ (x_1^3, x_2^3; \alpha_3^x, \alpha_3^y) \end{bmatrix} = \begin{bmatrix} (2, 2; 2, 2) \\ (0.8333, 3.1667; 1, 1) \\ (0.5, 0.5; 1, 1) \end{bmatrix},$$
then \( A\breve{X}_v = \begin{bmatrix} (b_1^1, b_2^1; \alpha_1^1, \alpha_1^1) \\ (b_1^2, b_2^2; \alpha_1^2, \alpha_1^2) \\ (b_1^3, b_2^3; \alpha_1^3, \alpha_1^3) \end{bmatrix} = \begin{bmatrix} (-3.8334, 0.8334; 5, 5) \\ (-4.6667, -2.3333; 4, 4) \\ (-2.6667, -0.3333; 6, 6) \end{bmatrix}, \)

\[ D_1(A\breve{X}_v, B) = 1.1666, \]
\[ D_2(A\breve{X}_v, B) = 0.763763. \]

In fact, there are many nearer (symmetric or non-symmetric) approximate solutions based on the distance metric function in Definition 1.8.

In this note, we illustrate two cases for approximate fuzzy solutions.

**Case 1: Symmetric approximate solution**

The following \( L-L \) fuzzy vector \( \breve{X}_1 \) is a symmetric approximate solution for the system, with distance metric function smaller than distance of solution \( \breve{X}_v \) in [1].

Given

\[ \breve{X}_1 = \begin{bmatrix} (2, 2; 1, 1) \\ (0.75, 3.25; 0.75, 0.75) \\ (0.5, 0.5; 2.5, 2.5) \end{bmatrix}, \] then \( A\breve{X}_1 = \begin{bmatrix} (-4, 1; 5, 5) \\ (-4.75, -2.25; 4.25, 4.25) \\ (-2.75, -0.25; 5.25, 5.25) \end{bmatrix}, \)

and the following result is obtained using Definition 1.8.

\[ D_1(A\breve{X}_1, \breve{B}) = 0.707107, \]
\[ D_2(A\breve{X}_1, \breve{B}) = 0.559017. \]

**Case 2: Non-symmetric approximate solution**

The following \( L-R \) fuzzy number vector \( \breve{X}_2 \) is a non-symmetric approximate solution for the system, with distance metric function smaller than distance of solution \( \breve{X}_v \) in [1].

Given

\[ \breve{X}_2 = \begin{bmatrix} \left( \frac{12}{7}, \frac{13}{7}; \frac{7}{7}, \frac{5}{7} \right) \\ \left( \frac{5}{2}, \frac{10}{7}; \frac{5}{7}, \frac{2}{7} \right) \\ \left( \frac{1}{2}, \frac{1}{2}; \frac{5}{7}, \frac{5}{7} \right) \end{bmatrix}, \] then \( A\breve{X}_2 = \begin{bmatrix} (-4, 1; 5, 5) \\ (-5, -\frac{5}{4}; 4, \frac{9}{4}) \\ (-3, -\frac{1}{2}; 5, \frac{11}{2}) \end{bmatrix}, \)
and we produce the following results

\[ D_1(\hat{A}\hat{X}_2, \hat{B}) = 0.809017, \]
\[ D_2(\hat{A}\hat{X}_2, \hat{B}) = 0.612372. \]

**Note:**
Our new solutions are obtained by using distance metric function which not only provides \(L-L\) fuzzy number vector, but also \(L-R\) fuzzy number vector.

**References**


