ERRATUM TO “NEW APPROACH FOR CLOSURE SPACES BY RELATIONS”

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Abstract. In this note, an alleged lemma 3.6 stated in [2] is incorrect in general, by giving an example. In addition to this point, if the closure space studied in [2] was $T_1$ space, then it is the discrete space $(X, P(X))$. As a consequence, Proposition 6.4, Corollary 6.4, Proposition 6.5, Corollary 6.6, Proposition 6.6 and Corollary 6.7 mentioned in [2] are trivially satisfied without proof.

1. Introduction

Definition 1. [1] Let $X$ be a nonempty set and $R$ be a binary relation on $X$. The minimal neighbourhood of $x \in X$ is defined as:

$$\langle x \rangle_R = \cap \{pR : x \in pR\},$$

where $pR = \{q \in X : (p, q) \in R\}$.

Definition 2. [2] Let $R$ be a binary relation on a nonempty set $X$. The closure operation on $X$, denoted by $cl_R$, defined as follows:

$$cl_R(A) = A \cup \{x \in X : \langle x \rangle_R \cap A \neq \emptyset\}.$$ 

Theorem 1. [2] Let $R$ be a binary relation on a nonempty set $X$. Then a closure space $(X, cl_R)$ is an Alexandrov topological space.

Lemma 1. [3] Let $(X, \tau)$ be an Alexandrov $T_1$-space. Then $(X, \tau)$ is the discrete space; that is, $\tau = P(X)$.

Lemma 3.6 in [2] claimed that for any binary relation $R$ on $X$ the following implication has been satisfied:

$$x \in cl_R(\{y\}) \Rightarrow y \in \langle x \rangle_R.$$

This assertion is wrong in general by giving example.

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2. Main results

The following example shows that the sufficient condition of Lemma 3.6 in [2] is incorrect in general.

Example 1. Let \( X = \{a, b, c, d\} \) and \( R = \{(a, a), (a, b), (b, c), (d, a)\} \). Then \( \langle a \rangle_R = \{a\}, \langle b \rangle_R = \{a, b\}, \langle c \rangle_R = \{c\} \) and \( \langle d \rangle_R = \emptyset \). It's clear that \( d \in \text{cl}_R(\{d\}) = \{d\} \), but \( d \notin \langle d \rangle_R \).

**Proposition 1.** Let \( R \) be a binary relation on a nonempty set \( X \). Then any closure space \( (X, \text{cl}_R) \) which is \( T_1 \) is the discrete space \( (X, P(X)) \).

**Proof.** The result is a direct consequence of Theorem 1 and Lemma 1. \( \square \)

**Remark 1.** It should be noted that Proposition 1 implies that Proposition 6.4, Corollary 6.4, Proposition 6.5, Corollary 6.6, Proposition 6.6 and Corollary 6.7 stated in [2] are trivially satisfied without proof.

**REFERENCES**


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