

COMPLETE SPACELIKE HYPERSURFACES WITH CONSTANT SCALAR CURVATURE

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ABSTRACT. In this paper, we characterize the n -dimensional ($n \geq 3$) complete spacelike hypersurfaces M^n in a de Sitter space S_1^{n+1} with constant scalar curvature and with two distinct principal curvatures one of which is simple. We show that M^n is a locus of moving $(n-1)$ -dimensional submanifold $M_1^{n-1}(s)$, along $M_1^{n-1}(s)$ the principal curvature λ of multiplicity $n-1$ is constant and $M_1^{n-1}(s)$ is umbilical in S_1^{n+1} and is contained in an $(n-1)$ -dimensional sphere $S^{n-1}(c(s)) = E^n(s) \cap S_1^{n+1}$ and is of constant curvature $\left(\frac{d\{\log|\lambda^2-(1-R)|^{1/n}\}}{ds}\right)^2 - \lambda^2 + 1$, where s is the arc length of an orthogonal trajectory of the family $M_1^{n-1}(s)$, $E^n(s)$ is an n -dimensional linear subspace in R_1^{n+2} which is parallel to a fixed $E^n(s_0)$ and $u = |\lambda^2 - (1-R)|^{-\frac{1}{n}}$ satisfies the ordinary differential equation of order 2, $\frac{d^2u}{ds^2} - u\left(\pm \frac{n-2}{2} \frac{1}{u^n} + R - 2\right) = 0$.

1. INTRODUCTION

Let R_1^{n+2} be the $(n+2)$ -dimensional Lorentz-Minkowski space and S_1^{n+1} be the de Sitter space given by $S_1^{n+1} = \{p \in R_1^{n+2} \mid \langle p, p \rangle = 1\}$. A hypersurface M^n of S_1^{n+1} is said to be spacelike if the induced metric on M^n from that of ambient space is positive definite. In [4] we investigated the spacelike hypersurfaces M^n in a de Sitter space S_1^{n+1} with constant scalar curvature and with two distinct principal curvatures whose multiplicities are greater than 1. We showed that

Theorem 1.1 ([4]). *Let M^n be an n -dimensional complete spacelike hypersurface in S_1^{n+1} with constant scalar curvature and with two distinct principal curvatures. If the multiplicities of these two distinct principal curvatures are greater than 1, then M^n is isometric to the Riemannian product $H^k(\sinh r) \times S^{n-k}(\cosh r)$, $1 < k < n-1$.*

As we know that Otsuki [3] characterized the minimal hypersurfaces in a Riemannian manifold \bar{M} of constant curvature \bar{c} with two distinct principal curvatures

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one of which is simple and Cheng [2] investigated the n -dimensional oriented complete hypersurfaces ($n \geq 3$) in Euclidean space R^{n+1} with constant scalar curvature and with two distinct principal curvatures one of which is simple. It is natural and important to investigate the spacelike hypersurfaces M^n in a de Sitter space S_1^{n+1} with constant scalar curvature and with two distinct principal curvatures one of which is simple. In this paper, we obtain the following

Theorem 1.2. *Let M^n be an n -dimensional ($n \geq 3$) complete spacelike hypersurface in a de Sitter space S_1^{n+1} with constant scalar curvature $n(n-1)R$ and with two distinct principal curvatures one of which is simple, then M^n is a locus of moving $(n-1)$ -dimensional submanifold $M_1^{n-1}(s)$, along $M_1^{n-1}(s)$ the principal curvature λ of multiplicity $n-1$ is constant and $M_1^{n-1}(s)$ is umbilical in S_1^{n+1} and is contained in an $(n-1)$ -dimensional sphere $S^{n-1}(c(s)) = E^n(s) \cap S_1^{n+1}$ and is of constant curvature $(\frac{d\{\log|\lambda^2-(1-R)|^{1/n}\}}{ds})^2 - \lambda^2 + 1$, where s is the arc length of an orthogonal trajectory of the family $M_1^{n-1}(s)$, $E^n(s)$ is an n -dimensional linear subspace in R_1^{n+2} which is parallel to a fixed $E^n(s_0)$ and $u = |\lambda^2 - (1-R)|^{-\frac{1}{n}}$ satisfies the ordinary differential equation of order 2*

$$\frac{d^2u}{ds^2} - u\left(\pm \frac{n-2}{2} \frac{1}{u^n} + R - 2\right) = 0.$$

2. PRELIMINARIES

Let M^n be an n -dimensional spacelike hypersurfaces in S_1^{n+1} , we choose a local field of semi-Riemannian orthonormal frames e_1, \dots, e_{n+1} in S_1^{n+1} such that at each point of M^n , e_1, \dots, e_n span the tangent space of M^n and form an orthonormal frame there. We use the following convention on the range of indices:

$$1 \leq A, B, C, \dots \leq n+1; \quad 1 \leq i, j, k, \dots \leq n.$$

Let $\omega_1, \dots, \omega_{n+1}$ be the dual frame field so that the semi-Riemannian metric of S_1^{n+1} is given by $d\bar{s}^2 = \sum_i \omega_i^2 - \omega_{n+1}^2 = \sum_A \epsilon_A \omega_A^2$, where $\epsilon_i = 1$ and $\epsilon_{n+1} = -1$.

The structure equations of S_1^{n+1} are given by

$$(2.1) \quad d\omega_A = \sum_B \epsilon_B \omega_{AB} \wedge \omega_B, \quad \omega_{AB} + \omega_{BA} = 0,$$

$$(2.2) \quad d\omega_{AB} = \sum_C \epsilon_C \omega_{AC} \wedge \omega_{CB} + \Omega_{AB},$$

where

$$(2.3) \quad \Omega_{AB} = -\frac{1}{2} \sum_{C,D} K_{ABCD} \omega_C \wedge \omega_D,$$

$$(2.4) \quad K_{ABCD} = \epsilon_A \epsilon_B (\delta_{AC} \delta_{BD} - \delta_{AD} \delta_{BC}).$$

Restrict these forms to M^n , we have

$$(2.5) \quad \omega_{n+1} = 0.$$

Cartan's Lemma implies that

$$(2.6) \quad \omega_{n+1i} = \sum_j h_{ij} \omega_j, \quad h_{ij} = h_{ji}.$$

The structure equations of M^n are

$$(2.7) \quad d\omega_i = \sum_j \omega_{ij} \wedge \omega_j, \quad \omega_{ij} + \omega_{ji} = 0,$$

$$(2.8) \quad d\omega_{ij} = \sum_k \omega_{ik} \wedge \omega_{kj} - \frac{1}{2} \sum_{k,l} R_{ijkl} \omega_k \wedge \omega_l,$$

$$(2.9) \quad R_{ijkl} = (\delta_{ik} \delta_{jl} - \delta_{il} \delta_{jk}) - (h_{ik} h_{jl} - h_{il} h_{jk}),$$

where R_{ijkl} are the components of the curvature tensor of M^n and

$$(2.10) \quad h = \sum_{i,j} h_{ij} \omega_i \otimes \omega_j$$

is the second fundamental form of M^n .

From the above equation, we have

$$(2.11) \quad n(n-1)R = n(n-1) - n^2 H^2 + |h|^2,$$

where $n(n-1)R$ is the scalar curvature of M^n , H is the mean curvature, and $|h|^2 = \sum_{i,j} h_{ij}^2$ is the squared norm of the second fundamental form of M^n .

The Codazzi equation and the Ricci identity are

$$(2.12) \quad h_{ijk} = h_{ikj},$$

$$(2.13) \quad h_{ijkl} - h_{ijlk} = \sum_m h_{mj} R_{mikl} + \sum_m h_{im} R_{mjkl},$$

where h_{ijk} and h_{ijkl} denote the first and the second covariant derivatives of h_{ij} .

We choose e_1, \dots, e_n such that $h_{ij} = \lambda_i \delta_{ij}$. From (2.6) we have

$$(2.14) \quad \omega_{n+1i} = \lambda_i \omega_i, \quad i = 1, 2, \dots, n.$$

Hence, we have from the structure equations of M^n

$$(2.15) \quad \begin{aligned} d\omega_{n+1i} &= d\lambda_i \wedge \omega_i + \lambda_i d\omega_i \\ &= d\lambda_i \wedge \omega_i + \lambda_i \sum_j \omega_{ij} \wedge \omega_j. \end{aligned}$$

On the other hand, we have on the curvature forms of S_1^{n+1} ,

$$(2.16) \quad \begin{aligned} \Omega_{n+1i} &= -\frac{1}{2} \sum_{C,D} K_{n+1iCD} \omega_C \wedge \omega_D \\ &= \frac{1}{2} \sum_{C,D} (\delta_{n+1C} \delta_{iD} - \delta_{n+1D} \delta_{iC}) \omega_C \wedge \omega_D \\ &= \omega_{n+1} \wedge \omega_i = 0. \end{aligned}$$

Therefore, from the structure equations of S_1^{n+1} , we have

$$(2.17) \quad \begin{aligned} d\omega_{n+1i} &= \sum_j \omega_{n+1j} \wedge \omega_{ji} - \omega_{n+1n+1} \wedge \omega_{n+1i} + \Omega_{n+1i} \\ &= \sum_j \lambda_j \omega_{ij} \wedge \omega_j. \end{aligned}$$

From (2.15) and (2.17), we obtain

$$(2.18) \quad d\lambda_i \wedge \omega_i + \sum_j (\lambda_i - \lambda_j) \omega_{ij} \wedge \omega_j = 0.$$

Putting

$$(2.19) \quad \psi_{ij} = (\lambda_i - \lambda_j) \omega_{ij}.$$

Then $\psi_{ij} = \psi_{ji}$. (2.18) can be written as

$$(2.20) \quad \sum_j (\psi_{ij} + \delta_{ij} d\lambda_j) \wedge \omega_j = 0.$$

By E. Cartan's Lemma, we get

$$(2.21) \quad \psi_{ij} + \delta_{ij} d\lambda_j = \sum_k Q_{ijk} \omega_k,$$

where Q_{ijk} are uniquely determined functions such that

$$(2.22) \quad Q_{ijk} = Q_{ikj}.$$

3. PROOF OF THEOREM

We firstly have the following Proposition 3.1 due to [1], which original due to Otsuki [3] for Riemannian space forms.

Proposition 3.1 ([1]). *Let M^n be a spacelike hypersurface in S_1^{n+1} such that the multiplicities of the principal curvatures are constant. Then the distribution of the space of principal vectors corresponding to each principal curvature λ is completely integrable. In particular, if the multiplicity of a principal curvature is greater than 1, then this principal curvature is constant on each integral submanifold of the corresponding distribution of the space of the principal vectors.*

Proof of Theorem 1.2. Let M^n be an n -dimensional complete spacelike hypersurface with constant scalar curvature and with two distinct principal curvatures one of which is simple, that is, without lose of generality, we may assume

$$\lambda_1 = \lambda_2 = \cdots = \lambda_{n-1} = \lambda, \quad \lambda_n = \mu,$$

where λ_i for $i = 1, 2, \dots, n$ are the principal curvatures of M^n . Since the scalar curvature $n(n-1)R$ is constant, from (2.11), we obtain

$$(3.1) \quad n(n-1)(1-R) = (n-1)(n-2)\lambda^2 + 2(n-1)\lambda\mu.$$

If $\lambda = 0$ at some points, then $R = 1$ at these points from (3.1), since R is constant, we know $R = 1$ on M^n . Since these principal curvatures λ and μ are continuous on M^n , from (3.1) and $R = 1$ we obtain $\lambda = 0$ on M^n . Hence, from the Gauss

equation, the sectional curvature of M^n $R_{ijij} = 1 - \lambda\mu = 1 > 0$, by Myers' theorem we know that M^n is compact. From the result of Zheng [6, 5], we know that M^n is a totally umbilical spacelike hypersurface. This is impossible because we assumed that M^n is of two distinct principal curvatures. Hence, we can assume $\lambda \neq 0$ on M^n . From (3.1), we have

$$(3.2) \quad \mu = \frac{n(1 - R)}{2\lambda} - \frac{(n - 2)\lambda}{2}.$$

Therefore, we get

$$\lambda - \mu = n \frac{\lambda^2 - (1 - R)}{2\lambda} \neq 0,$$

we know $\lambda^2 - (1 - R) \neq 0$.

Let $u = |\lambda^2 - (1 - R)|^{-\frac{1}{n}}$. We denote the integral submanifold through $x \in M^n$ corresponding to λ by $M_1^{n-1}(x)$. Putting

$$(3.3) \quad d\lambda = \sum_{k=1}^n \lambda_{,k} \omega_k, \quad d\mu = \sum_{k=1}^n \mu_{,k} \omega_k.$$

From Proposition 3.1, we have

$$(3.4) \quad \lambda_{,1} = \lambda_{,2} = \dots = \lambda_{,n-1} = 0 \quad \text{on} \quad M_1^{n-1}(x).$$

From (3.2), we have

$$(3.5) \quad d\mu = \left[-\frac{n(1 - R)}{2\lambda^2} - \frac{n - 2}{2} \right] d\lambda.$$

Hence, we also have

$$(3.6) \quad \mu_{,1} = \mu_{,2} = \dots = \mu_{,n-1} = 0 \quad \text{on} \quad M_1^{n-1}(x).$$

In this case, we may consider locally λ is a function of the arc length s of the integral curve of the principal vector field e_n corresponding to the principal curvature μ . From (2.21) and (3.4), we have for $1 \leq j \leq n - 1$,

$$(3.7) \quad \begin{aligned} d\lambda &= d\lambda_j = \sum_{k=1}^n Q_{jjk} \omega_k \\ &= \sum_{k=1}^{n-1} Q_{jjk} \omega_k + Q_{jjn} \omega_n = \lambda_{,n} \omega_n. \end{aligned}$$

Therefore, we have

$$(3.8) \quad Q_{jjk} = 0, \quad 1 \leq k \leq n - 1, \quad \text{and} \quad Q_{jjn} = \lambda_{,n}.$$

By (2.21) and (3.6), we have

$$(3.9) \quad \begin{aligned} d\mu &= d\lambda_n = \sum_{k=1}^n Q_{nnk} \omega_k \\ &= \sum_{k=1}^{n-1} Q_{nnk} \omega_k + Q_{nnn} \omega_n = \sum_{i=1}^n \mu_{,i} \omega_i = \mu_{,n} \omega_n. \end{aligned}$$

Hence, we obtain

$$(3.10) \quad Q_{nnk} = 0, \quad 1 \leq k \leq n-1, \quad \text{and} \quad Q_{nnn} = \mu_{,n}.$$

From (3.5), we get

$$(3.11) \quad Q_{nnn} = \mu_{,n} = \left[-\frac{n(1-R)}{2\lambda^2} - \frac{n-2}{2} \right] \lambda_{,n}.$$

From the definition of ψ_{ij} , if $i \neq j$, we have $\psi_{ij} = 0$ for $1 \leq i \leq n-1$ and $1 \leq j \leq n-1$. Therefore, from (2.21), if $i \neq j$ and $1 \leq i \leq n-1$ and $1 \leq j \leq n-1$ we have

$$(3.12) \quad Q_{ijk} = 0, \quad \text{for any } k.$$

By (2.21), (3.8), (3.10), (3.11) and (3.12), we get

$$(3.13) \quad \begin{aligned} \psi_{jn} &= \sum_{k=1}^n Q_{jnk} \omega_k \\ &= Q_{jjn} \omega_j + Q_{jnn} \omega_n = \lambda_{,n} \omega_j. \end{aligned}$$

Since λ and μ are two distinct principal curvatures of M^n , we have

$$\lambda - \mu = n \frac{\lambda^2 - (1-R)}{2\lambda} \neq 0.$$

From (2.19), (3.2) and (3.13) we have

$$(3.14) \quad \begin{aligned} \omega_{jn} &= \frac{\psi_{jn}}{\lambda - \mu} = \frac{\lambda_{,n}}{\lambda - \mu} \omega_j \\ &= \frac{\lambda_{,n}}{\lambda - \left[\frac{n(1-R)}{2\lambda} - \frac{n-2}{2} \lambda \right]} \omega_j \\ &= \frac{2\lambda\lambda_{,n}}{n[\lambda^2 - (1-R)]} \omega_j. \end{aligned}$$

Therefore, from the structure equations of M^n we have

$$d\omega_n = \sum_{k=1}^{n-1} \omega_k \wedge \omega_{kn} + \omega_{nn} \wedge \omega_n = 0.$$

Therefore, we may put $\omega_n = ds$. By (3.7) and (3.9), we get

$$d\lambda = \lambda_{,n} ds, \quad \lambda_{,n} = \frac{d\lambda}{ds},$$

and

$$d\mu = \mu_{,n} ds, \quad \mu_{,n} = \frac{d\mu}{ds}.$$

Then we have

$$(3.15) \quad \begin{aligned} \omega_{jn} &= \frac{2\lambda\lambda_{,n}}{n[\lambda^2 - (1 - R)]} \omega_j = \frac{2\lambda \frac{d\lambda}{ds}}{n[\lambda^2 - (1 - R)]} \omega_j \\ &= \frac{d\{\log |\lambda^2 - (1 - R)|^{\frac{1}{n}}\}}{ds} \omega_j. \end{aligned}$$

(3.15) shows that the integral submanifold $M_1^{n-1}(x)$ corresponding to λ and s is umbilical in M^n and S_1^{n+1} .

From (3.15) and the structure equations of S_1^{n+1} , we have

$$\begin{aligned} d\omega_{jn} &= \sum_{k=1}^{n-1} \omega_{jk} \wedge \omega_{kn} + \omega_{jn} \wedge \omega_{nn} + \omega_{jn+1} \wedge \omega_{n+1n} + \Omega_{jn} \\ &= \sum_{k=1}^{n-1} \omega_{jk} \wedge \omega_{kn} + \omega_{jn+1} \wedge \omega_{n+1n} - \omega_j \wedge \omega_n \\ &= \frac{d\{\log |\lambda^2 - (1 - R)|^{\frac{1}{n}}\}}{ds} \sum_{k=1}^{n-1} \omega_{jk} \wedge \omega_k - (\lambda\mu + 1)\omega_j \wedge ds. \end{aligned}$$

From (3.15) we have

$$\begin{aligned} d\omega_{jn} &= \frac{d^2\{\log |\lambda^2 - (1 - R)|^{\frac{1}{n}}\}}{ds^2} ds \wedge \omega_j + \frac{d\{\log |\lambda^2 - (1 - R)|^{\frac{1}{n}}\}}{ds} d\omega_j \\ &= \frac{d^2\{\log |\lambda^2 - (1 - R)|^{\frac{1}{n}}\}}{ds^2} ds \wedge \omega_j + \frac{d\{\log |\lambda^2 - (1 - R)|^{\frac{1}{n}}\}}{ds} \sum_{k=1}^n \omega_{jk} \wedge \omega_k \\ &= \left\{ -\frac{d^2\{\log |\lambda^2 - (1 - R)|^{\frac{1}{n}}\}}{ds^2} + \left[\frac{d\{\log |\lambda^2 - (1 - R)|^{\frac{1}{n}}\}}{ds} \right]^2 \right\} \omega_j \wedge ds \\ &\quad + \frac{d\{\log |\lambda^2 - (1 - R)|^{\frac{1}{n}}\}}{ds} \sum_{k=1}^{n-1} \omega_{jk} \wedge \omega_k. \end{aligned}$$

From the above two equalities, we have

$$(3.16) \quad \frac{d^2\{\log |\lambda^2 - (1 - R)|^{\frac{1}{n}}\}}{ds^2} - \left\{ \frac{d\{\log |\lambda^2 - (1 - R)|^{\frac{1}{n}}\}}{ds} \right\}^2 - (\lambda\mu + 1) = 0.$$

From (3.2) we get

$$(3.17) \quad \begin{aligned} \frac{d^2\{\log |\lambda^2 - (1 - R)|^{\frac{1}{n}}\}}{ds^2} - \left\{ \frac{d\{\log |\lambda^2 - (1 - R)|^{\frac{1}{n}}\}}{ds} \right\}^2 \\ + \frac{n-2}{2} [\lambda^2 - (1 - R)] + R - 2 = 0. \end{aligned}$$

Since we define $u = |\lambda^2 - (1 - R)|^{-\frac{1}{n}}$, we obtain from the above equation

$$(3.18) \quad \frac{d^2u}{ds^2} - u \left(\pm \frac{n-2}{2} \frac{1}{u^n} + R - 2 \right) = 0.$$

Since S_1^{n+1} is an $(n+1)$ -dimensional de Sitter space of constant 1 in R_1^{n+2} . We consider the frame $e_1, e_2, \dots, e_n, e_{n+1}, e_{n+2}$ in R_1^{n+2} . Since the second fundamental form of S_1^{n+1} as the hypersurface R_1^{n+2} is given by $h_{AB} = -\sum_B \epsilon_B \delta_{AB}$, we have

$$\omega_{n+1n+2} = 0, \quad \text{and} \quad \omega_{in+2} = -\omega_i.$$

Then, from (2.14), (3.15) and (3.16), we have

$$\begin{aligned} de_i &= \sum_{j=1}^{n-1} \omega_{ij} e_j + \omega_{in} e_n + \omega_{in+1} e_{n+1} + \omega_{in+2} e_{n+2} \\ &= \sum_{j=1}^{n-1} \omega_{ij} e_j + \frac{d\{\log|\lambda^2 - (1-R)|^{\frac{1}{n}}\}}{ds} \omega_i e_n - \lambda \omega_i e_{n+1} - e_{n+2} \omega_i \\ &= \sum_{j=1}^{n-1} \omega_{ij} e_j + \left[\frac{d\{\log|\lambda^2 - (1-R)|^{\frac{1}{n}}\}}{ds} e_n - \lambda e_{n+1} - e_{n+2} \right] \omega_i. \end{aligned}$$

On the other hand, by means of (3.16) we get

$$\begin{aligned} d\left\{ \frac{d\{\log|\lambda^2 - (1-R)|^{\frac{1}{n}}\}}{ds} e_n - \lambda e_{n+1} - e_{n+2} \right\} &= d\left\{ \frac{d\{\log|\lambda^2 - (1-R)|^{\frac{1}{n}}\}}{ds} \right\} e_n \\ &\quad + \frac{d\{\log|\lambda^2 - (1-R)|^{\frac{1}{n}}\}}{ds} de_n - d\lambda e_{n+1} - \lambda de_{n+1} - de_{n+2} \\ &= \left\{ \frac{d^2\{\log|\lambda^2 - (1-R)|^{\frac{1}{n}}\}}{ds^2} e_n - \frac{d\lambda}{ds} e_{n+1} \right\} ds \\ &\quad + \frac{d\{\log|\lambda^2 - (1-R)|^{\frac{1}{n}}\}}{ds} \left(\sum_{j=1}^{n-1} \omega_{nj} e_j + \omega_{nn+1} e_{n+1} + \omega_{nn+2} e_{n+2} \right) \\ &\quad - \lambda \left(\sum_{j=1}^{n-1} \omega_{n+1j} e_j + \omega_{n+1n} e_n + \omega_{n+1n+2} e_{n+2} \right) - \sum_{j=1}^{n-1} \omega_i e_j - \omega_n e_n \\ &= \left\{ \frac{d^2\{\log|\lambda^2 - (1-R)|^{\frac{1}{n}}\}}{ds^2} e_n - \frac{d\lambda}{ds} e_{n+1} \right\} ds \\ &\quad + \frac{d\{\log|\lambda^2 - (1-R)|^{\frac{1}{n}}\}}{ds} \left(\sum_{j=1}^{n-1} \omega_{nj} e_j - \mu \omega_n e_{n+1} - \omega_n e_{n+2} \right) \\ &\quad - \lambda \left(\lambda \sum_{j=1}^{n-1} \omega_j e_j + \mu \omega_n e_n \right) - \sum_{j=1}^{n-1} \omega_i e_j - \omega_n e_n \\ &\equiv \left[\frac{d^2\{\log|\lambda^2 - (1-R)|^{\frac{1}{n}}\}}{ds^2} - \lambda \mu - 1 \right] e_n \omega_n \\ &\quad - \left\{ \frac{d\lambda}{ds} + \frac{d\{\log|\lambda^2 - (1-R)|^{\frac{1}{n}}\}}{ds} \mu \right\} e_{n+1} \omega_n \end{aligned}$$

$$\begin{aligned}
 & - \frac{d\{\log|\lambda^2 - (1 - R)|^{\frac{1}{n}}\}}{ds} e_{n+2}\omega_n \quad (\text{mod}\{e_1, \dots, e_{n-1}\}) \\
 & = \frac{d\{\log|\lambda^2 - (1 - R)|^{\frac{1}{n}}\}}{ds} \left\{ \frac{d\{\log|\lambda^2 - (1 - R)|^{\frac{1}{n}}\}}{ds} e_n - \lambda e_{n+1} - e_{n+2} \right\} ds.
 \end{aligned}$$

We put

$$W = e_1 \wedge \dots \wedge e_{n-1} \wedge \left\{ \frac{d\{\log|\lambda^2 - (1 - R)|^{\frac{1}{n}}\}}{ds} e_n - \lambda e_{n+1} - e_{n+2} \right\}.$$

Therefore we have

$$(3.19) \quad dW = \frac{d\{\log|\lambda^2 - (1 - R)|^{\frac{1}{n}}\}}{ds} W ds,$$

(3.19) shows that n -vector W in R_1^{n+2} is constant along $M_1^{n-1}(x)$. Hence, there exists an n -dimensional linear subspace $E^n(s)$ in R_1^{n+2} containing $M_1^{n-1}(x)$. By (3.19), the n -vector field W depends only on s and by integrating it, we get

$$W = \left\{ \frac{\lambda^2(s) - (1 - R)}{\lambda^2(s_0) - (1 - R)} \right\}^{\frac{1}{n}} = W(s_0).$$

Hence, we have that $E^n(s)$ is parallel to $E^n(s_0)$ in R_1^{n+2} .

Since $\Omega_{ij} = -\omega_i \wedge \omega_j$, from (2.2) the curvature of $M_1^{n-1}(x)$ is given by

$$\begin{aligned}
 d\omega_{ij} - \sum_{k=1}^{n-1} \omega_{ik} \wedge \omega_{kj} & = \omega_{in} \wedge \omega_{nj} - \omega_{in+1} \wedge \omega_{n+1j} - \omega_i \wedge \omega_j \\
 & = - \left\{ \left(\frac{d\{\log|\lambda^2 - (1 - R)|^{\frac{1}{n}}\}}{ds} \right)^2 - \lambda^2 + 1 \right\} \omega_i \wedge \omega_j.
 \end{aligned}$$

Therefore we know that the curvature of $M_1^{n-1}(x)$ is $\left(\frac{d\{\log|\lambda^2 - (1 - R)|^{\frac{1}{n}}\}}{ds} \right)^2 - \lambda^2 + 1$ and $M_1^{n-1}(x)$ is contained in an $(n - 1)$ -dimensional sphere $S^{n-1}(c(s)) = E^n(s) \cap S_1^{n+1}$ of the intersection of S_1^{n+1} and an n -dimensional linear subspace $E^n(s)$ in R_1^{n+2} which is parallel to a fixed $E^n(s_0)$. This completes the proof of Theorem 1.2.

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