1) General description of the school system

1.1 Compulsory education in Luxembourg is divided into two levels: primary education from age 6 to 12 and secondary education from age 12 to 19. However, pupils may leave at the age of 15 when compulsory education ends.

The primary schools, which are located near the children's homes, offer the same education for everyone. Curriculum and timetable are fixed by the Ministry of Education. The compulsory subjects include foreign languages (German and French), calculus, history, biology and geography.

There are two types of secondary schools: the classical grammar schools (Lycées classiques) and the technical/vocational schools (Lycées techniques). The classical schools prepare the pupils for further studies at a university, whereas the technical education mainly aims at preparing the students for their entrance to the employment market. In the final year of primary school (age 12), the children, together with their parents, have to choose between classical and technical education. A team of primary and secondary teachers and a psychologist guide them in their choice.

At the end of secondary education (age 19) pupils have to take the secondary school leaving examination (examen de fin d'études secondaires) organised by the Ministry of Education. Secondary education is divided into three "cycles": a lower cycle (years 1, 2, 3) compulsory for all pupils, a general cycle of the higher division (years 4, 5) and a specialisation cycle (years 6, 7).

The national curriculum of the lower cycle includes foreign languages (German, French, English, and Latin for those doing classics), mathematics, history and natural sciences. The subjects in the higher division of classical and technical secondary education are very different.

In classical education the general cycle (years 4 and 5) proposes two orientations: "orientation littéraire and orientation scientifique". Pupils of the general cycle have to learn three or four modern foreign languages, mathematics, history and natural sciences.
The specialisation cycle (years 6 and 7) offers seven "sections": a literary section, a mathematics section, a science and mathematics section, a social sciences and economics section, a music section and an arts section.

In technical education the higher division (years 4-7) is split up into three "régimes":
1)"régime technique" (upper level / years 4-7 / national school-leaving examination)
2)"régime de la formation de techniciens" (lower level / years 4-7 / national school-leaving examination)
3)"régime professionnel" (not full–time / two or three years). The curricula for for the upper division of technical education include languages, mathematics and computer science.

1.2 Place and importance of mathematics in the curriculum

In primary and in secondary education mathematics is compulsory for all pupils. The vehicular language of calculus in primary education and in the "régime professionnel" of technical secondary education is German. In classical secondary education and the upper level "régimes" of technical education the teachers speak French. From age 12 to 16 pupils spend three to four hours per week on mathematics (out of a total of thirty lessons). However, in the lower (professional) level of technical secondary education there are only one or two lessons of mathematics per week. Curriculum and textbooks fixed by the Ministry of Education are compulsory for secondary schools, even for the few private ones. There are periodic written tests (devoir écrits) (set and marked) proposed and assessed by the teacher of mathematics of the class.

2) Main objectives in mathematics

In the following, the study focuses on the mathematical education of 12 to 16-year-old children.

At classical secondary schools mathematics is considered as part of general culture and the main objective of the course is to cultivate the mind. Other objectives of mathematical education are:

- to develop children's faculties of abstraction, imagination and intuition,
- to train pupils in analysing, understanding and solving problems,
- to teach them to use the mathematical symbols correctly
- to do mathematics with pleasure by working regularly and with perseverance,
- to give an insight into the applications of mathematics in other sciences and into the history of mathematics,
- to make the children memorise definitions, theorems and proofs.

In technical secondary education the aims are different. The first objectives are the repetition of the mathematical notions seen in the past in order to generalise them, to
introduce new notions by concrete examples, to develop the skill of calculus and to train the use of a pocket calculator.

3) Basic contents

Every year the list of compulsory textbooks, the programme and the targets to be attained are fixed by the Ministry of Education in the "Horaires et Programmes."

In classical education the basic contents for 12 to 16-year-old children can be divided into three fields:

*SETS*–Venn diagram of 2 or 3 sets, finite and infinite sets, intersection and union of two or several sets, Cartesian product, number sets (natural, rational, real numbers), relations between sets, functions and composition of functions.

Pupils should be able to use the symbols for finite and infinite sets correctly and to show the result of a succession of operations on a diagram in the case of finite sets or function composition.

*NUMBERS AND ALGEBRA*

- computation rules and order in the set of integers, decimals, rational and real numbers, absolute value, power laws, divisibility, HCF, LCM square root (definition and computation rules).
- development and factorisation of algebraic expressions, identities polynomials (degree, roots, division by x–a)
- first degree equations with one or two unknowns, systems of equations, inequations and systems of inequations, resolution of second-degree equations by factorisation.
- problem solving by equations.
- linear and affine functions.

Pupils should be able to simplify, factorise and give the numerical value of algebraic expressions with or without denominator, to solve equations and inequations containing absolute values and to draw the graph of an affine function.

*GEOMETRY*

- the plane: line, half–line, segment, angles, relative position of two lines, perpendicularity, middle perpendicular, bissectrix, triangle, quadrangle and circle.
- transformations: symmetry, translation, rotation.
- triangles: construction, congruency, properties of the mediatrices, bissectrices, heights and angles.
- Pythagorean theorem: proof and deduction of other proprieties of a rectangular triangle.
- Thales theorem: proof and applications.
- Vectors: sum, difference, multiplication by a real number and colinearity.
• trigonometry in a rectangular triangle.
• Cartesian plane: co-ordinates of a point or a vector, equation of a line.

At age 13 (second year of classical secondary education) pupils take their first steps in demonstrating. They should be able to deduct new properties from the established ones.

In technical secondary education the basic contents of the curricula for 12 to 16-year-old children are similar. However, there is a big difference in the way of teaching and of assessing. New definitions or properties are made clear by concrete examples. The pupils themselves discover the properties of a given figure in geometry and have to try to generalise. They are trained to test formulas by numerical application and thereby they get a good ability to use a pocket calculator.

In the following, we give the main differences between the technical and classical curricula.

SETS: only correct use of the symbols.

*NUMBERS*
in upper technical education: quadratic equations with applications

*GEOMETRY*
parabola, hyperbola, area and volume of three-dimensional figures.

In professional education, with few lessons a week, pupils should know the rules for operations, the rule of three, numerical proofs, and read statistical representations.

4) Exemplary topics

4.1 Quadratic equations
In classical secondary education the study of the quadratic function is a topic of the fifth class (IIe). Till age 16 a quadratic equation is to be solved by factorisation. In upper-level technical education pupils learn to solve second degree equations by using the formulas and they know that \( y=ax^2+b \) is the equation of a parabola. Usually pupils have no difficulty in understanding this matter.

4.2 Pythagorean theorem
This theorem is given and proved in the first years of secondary education in classical and technical schools. There are many applications and a lot of properties of the rectangular
triangles proved by this theorem. In the introduction of irrational numbers the theorem is used to visualise those numbers.

4.3 Similarity
Problems of similarity arise in the presentation of the Thales theorem and vector multiplication by a real number, but similarity is not explicitly mentioned in the curriculum. New textbooks together with new curricula from 2001 onwards will include similarity.

4.4 Word problems
Word problems occur as application of equation resolutions. Pupils have difficulties in understanding the text and translating a concrete situation into equations.

4.5 Percentages
Percentages appear in word problems and are a topic in professional education and the calculation of interests is not an easy task for most pupils.

5) Teacher training
Teachers of mathematics in secondary schools are required to complete a 4-year course in the mathematics section at university level, usually in Belgium, France or Switzerland (there is only a first cycle - 2 first years at undergraduate level - of post-secondary education in Luxembourg). Then the future teachers of mathematics are trained for three years, must write a scientific thesis, a pedagogical report and pass a practical examination (leçon modèle, leçon d'inspection, corrections) in a classical and in a technical school.