

Zbl 326.02050

Erdős, Paul; Hechler, S.H.

On maximal almost-disjoint families over singular cardinals. (In English)

Infinite finite Sets, Colloq. Honour Paul Erdős, Keszthely 1973, Colloq. Math. Soc. Janos Bolyai 10, 597-604 (1975).

[For the entire collection see Zbl 293.00009.]

κ is infinite and $|X| = \kappa$. A collection F of subsets of X is a κ -maximal almost-disjoint family (κ -MADF) if (i) for $y \in F$, $|Y| = \kappa$, (ii) if $Y, Z \in F$ then $|Y \cap Z| < \kappa$, (iii) if $S \subset X$ and $|S| = \kappa$ then there is a $Y \in F$ such that $|Y \cap S| = \kappa$. There are no κ -MADF's of cardinality κ if κ is regular. This paper concerns the case where κ is singular. Theorem (GCH). There exists a κ -MADF of cardinality κ iff κ is singular. Theorem. If κ is singular and $cf(\kappa) = \lambda$ then it is consistent with ZFC that there exist κ -MADF's of very cardinality $\mu \leq 2^\lambda$ except $\mu = \lambda$. Open Problem. Is it consistent with ZFC that there exists a singular κ for which there are no κ -MADF's of cardinality κ ? The paper contains several other theorems and problems.

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Classification:

03E55 Large cardinals

03E35 Consistency and independence results (set theory)